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1991 J. Phys. A: Math. Gen. 24 3427

(http://iopscience.iop.org/0305-4470/24/14/029)

J. Phys. A: Math. Gen. 24 (1991) 3427–3434. Printed in the UK

Quantization of null spinning strings with U(1) gauge symmetry

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Received 19 October 1990, in final form 18 February 1991

Abstract. We describe null spinning strings with U(1) gauge symmetry in terms of a phase-space Lagrangian and then quantize it in the light-cone gauge. With normal ordering we come to the conclusion that the critical dimension is $D=2$.

Strings with vanishing tension were first introduced by Schild [1]. He gave them the name 'null strings'. Recent research in the very high energy behaviour of string amplitudes [2–5] has triggered wide interest in the study of tensionless strings. Another reason why physicists are interested in tensionless strings is to simplify the theory by studying its zero-tension limit as an alternative approach to understanding the beauty and subtleties of string theory.

In constructing null string actions, the first difficulty is how to take the tension T to zero in a theory whose action is proportional to T. Several actions for tensionless strings have been introduced [6–10]; the significant common feature of these actions is the presence of auxiliary fields which are constrained in one way or another.

The quantization of null bosonic strings was investigated by Lizzi *et al* [11] who claimed that there are no critical dimensions for null strings, that the mass spectrum is continuous and the corresponding wavefunctions are plane waves. The difficulty that appeared in the above approach is that it is not clear why there are no critical dimensions. Gamboa *et al* [12, 13] showed that whether critical dimensions appear or not depends only on the ordering prescription which may lead to very different results. They considered the issue of the consistent quantization of null string theory with both Weyl operator ordering and normal operator ordering. The results is that with Weyl ordering the mass spectrum remains continuous and the critical dimensions do not exist, i.e. consistent with Lizzi's results, while with normal ordering the quantization is reasonable only in the critical dimensions: $D=26$ for bosonic strings and $D=10$ for spinning strings.

In this paper we consider the issue of the consistent quantization of null spinning strings with U(1) internal symmetry ($N=2$). We describe its action by means of a phase-space Lagrangian [8, 14, 15] where the limit $T\to 0$ can be taken in a consistent way. At a classical level, we consider its constraints and find that these constraints form a closed algebra which is analogous to the super-Virasoro algebra. Finally, with normal ordering, we quantize the $N=2$ null spinning string in the light-cone gauge. Our calculation gives the result of critical dimension $D=2$.

0305-4470/91/143427+08$03.50

We first briefly review the usual $N=2$ spinning string with non-vanishing tension T in order to establish notation and the usual constraint algebra. The action for $N=2$ tensile spinning strings is

$$S=-\frac{T}{2}\int d^2\sigma\sqrt{-g}[g^{mn}\partial_m X^\mu \partial_n \bar{X}_\mu - i\bar{\psi}_\mu \gamma^m \overleftrightarrow{\partial}_m \psi^\mu + A_m \bar{\psi}^\mu \gamma^m \psi_\mu$$
$$+(\partial_m X^\mu - \tfrac{1}{2}\bar{\chi}_m \psi^\mu)\bar{\psi}_\mu \gamma^n \gamma^m \chi_n + (\partial_m \bar{X}^\mu - \tfrac{1}{2}\bar{\psi}^\mu \chi_m)\bar{\chi}_n \gamma^m \gamma^n \psi_\mu]. \tag{1}$$

It is obvious that we cannot pursue the null string theory simply by letting the tension T in (1) approach zero. Note that there is no kinetic term for the U(1) gauge field A_m, world-sheet metric g_{mn} and gravitinos χ_m, $\bar{\chi}_m$. Therefore following the conventional procedure for the usual string with non-vanishing T the equations of motion of these fields imply vanishing of the following currents:

$$T_{mn}=-\frac{2}{\sqrt{-g}\,T}\frac{\delta S}{\delta g_{mn}}=0 \tag{2a}$$

$$J_m=-\frac{2}{\sqrt{-g}\,T}\frac{\delta S}{\delta \chi_m}=0 \tag{2b}$$

$$\bar{J}_m=-\frac{2}{\sqrt{-g}\,T}\frac{\delta S}{\delta \bar{\chi}_m}=0 \tag{2c}$$

$$V_m=-\frac{2}{\sqrt{-g}\,T}\frac{\delta S}{\delta A_m}=0. \tag{2d}$$

The Weyl invariance in two dimensions always allows us to choose a conformally flat metric. Besides, the supersymmetry and super-conformal symmetry can be used to set all the components of χ_m and $\bar{\chi}_m$ equal to zero [16]. In this gauge, we have

$$T_{mn}=\partial_m X^\mu \partial_n \bar{X}_\mu - \tfrac{1}{2}\eta_{mn}\eta^{kl}\partial_k X^\mu \partial_l \bar{X}_\mu - i\bar{\psi}_\mu \gamma_n \overleftrightarrow{\partial}_m \psi^\mu + i\eta_{mn}\bar{\psi}^\mu \gamma^k \overleftrightarrow{\partial}_k \psi_\mu \tag{3a}$$

$$J_m=\bar{\psi}^\mu \gamma_m \gamma^n \partial_n X_\mu \tag{3b}$$

$$\bar{J}_m=\partial_n \bar{X}_\mu \gamma^n \gamma_m \psi^\mu \tag{3c}$$

$$V_m=\bar{\psi}^\mu \gamma_m \psi_\mu \tag{3d}$$

and the Lagrangian obtained from (1) reduces in this gauge to

$$L=-\tfrac{1}{2}T(\eta^{mn}\partial_m X^\mu \partial_n \bar{X}_\mu - i\bar{\psi}^\mu \gamma^m \overleftrightarrow{\partial}_m \psi_\mu). \tag{4}$$

The canonical momenta are

$$P_\mu=\frac{\partial L}{\partial \dot{X}^\mu}=\frac{T}{2}\dot{\bar{X}}_\mu \tag{5a}$$

$$\bar{P}_\mu=\frac{\partial L}{\partial \dot{\bar{X}}^\mu}=\frac{T}{2}\dot{X}_\mu \tag{5b}$$

$$\pi_\mu=\frac{\partial L}{\partial \dot{\psi}^\mu}=-\frac{iT}{2}\psi^+_\mu \tag{6a}$$

$$\pi^+_\mu=\frac{\partial L}{\partial \dot{\psi}^{+\mu}}=\frac{iT}{2}\psi_\mu. \tag{6b}$$

Substituting equations (5) and (6) in equations (3), we obtain the constraints

$$H_{\perp}=P_{\mu}\bar{P}^{\mu}+\tfrac{1}{4}T^{2}(X^{\mu\prime}\bar{X}'_{\mu}-\mathrm{i}\psi_{\mu}^{+}\gamma^{5}\overleftrightarrow{\partial}_{\sigma}\psi^{\mu})=0 \tag{7a}$$

$$H_{\|}=\bar{P}^{\mu}\bar{X}'_{\mu}+P^{\mu}X'_{\mu}+\tfrac{1}{2}\mathrm{i}T\psi_{\mu}^{+}\overleftrightarrow{\partial}_{\sigma}\psi^{\mu}=0 \tag{7b}$$

$$S_{\alpha}=P_{\mu}\psi_{\alpha}^{\mu}-\tfrac{1}{2}T(\gamma^{5}\psi^{\mu})_{\alpha}\bar{X}'_{\mu}=0 \tag{7c}$$

$$\bar{S}_{\alpha}=\bar{P}_{\mu}\psi_{\alpha}^{+\mu}-\tfrac{1}{2}T(\psi^{+\mu}\gamma^{5})_{\alpha}X'_{\mu}=0 \tag{7d}$$

$$U=\psi_{\mu}^{+}\psi^{\mu}=0 \tag{7e}$$

where the bosonic constraints generate reparametrization transformations and U(1) gauge transformations, while the fermionic ones generate supersymmetry transformations.

The phasse-space Lagrangian [14] for the $N=2$ spinning string with non-vanishing tension is

$$L=\dot{X}^{\mu}P_{\mu}+\dot{\bar{X}}^{\mu}\bar{P}_{\mu}+\dot{\psi}_{\mu}^{+}\pi_{\mu}^{+}+\dot{\psi}_{\mu}\pi_{\mu}+\lambda_{\perp}H_{\perp}+\lambda_{\|}H_{\|}+\bar{\xi}_{\alpha}S_{\alpha}+\bar{S}_{\alpha}\xi_{\alpha}+\omega U. \tag{8}$$

The above structure follows from the fact that the string Hamiltonian is a pure constraint [13]. The bosonic Lagrange multipliers $\lambda_{\perp}$ and $\lambda_{\|}$ play the role of the independent components of the metric g_{mn}. Similarly, the fermionic ξ_{α} and $\bar{\xi}_{\alpha}$ replace the gravitino degrees of freedom. ω is the U(1) gauge degree of freedom.

The equations of motion for the Lagrange multipliers and for the conjugate momenta can be easily obtained from (8). Inserting them back into (8) again we obtain (1). Thus equations (1) and (8) are equivalent. However, (8) does not contain the tension T explicitly. Thus it allows us to consider the limit case of $T\to 0$ simply by means of setting T in the constraints (7) to zero. Therefore (8) serves as the appropriate Lagrangian for the null spinning string.

Setting the string tension equal to zero, the constraints (7) become

$$\begin{aligned}
&H_{\perp}=P^{\mu}\bar{P}_{\mu}=0\\
&H_{\|}=P^{\mu}X'_{\mu}+\bar{P}^{\mu}\bar{X}'_{\mu}=0\\
&S_{\alpha}=P_{\mu}\psi_{\alpha}^{\mu}=0\\
&\bar{S}_{\alpha}=\bar{P}_{\mu}\psi_{\alpha}^{+\mu}=0\\
&U=\psi_{\mu}^{+}\psi^{\mu}=0.
\end{aligned} \tag{9}$$

The relationships (6) between fermionic coordinates and momenta no longer exist. Instead, we have the constraints

$$\pi_{\alpha}^{\mu}=\pi_{\alpha}^{+\mu}=0 \tag{10}$$

which cannot be eliminated with the Dirac formalism of constrained systems [17]. Thus it is easy to see that the constraints (9) do not form a closed algebra. This difficulty for the $N=1$ case was circumvented in [13] by means of a singular reparametrization of the fields in the Lagrangian, so that at the end the result was a closed constraint algebra without second-class constraints and the fermionic variables evolute trivially. In our case, closure of constraint algebra can be enforced with the following elegent trick introduced by Barcelos-Neto *et al* [15].

Since the constraints $\pi_\alpha^\mu = \pi_\alpha^{+\mu} = 0$ cannot be eliminated with the Dirac formalism, we incorporate them in the relations (9) in the following way:

$$
\begin{aligned}
&H_\perp = P^\mu \bar{P}_\mu + a\pi_\mu \gamma^5 \psi^{\mu\prime} + b\psi_\mu^{+\prime}\gamma^5\pi^{+\mu} \\
&H_\parallel = P^\mu X'_\mu + \bar{P}^\mu \bar{X}'_\mu + c\pi^\mu \psi'_\mu + d\psi_\mu^{+\prime}\pi^{+\mu} \\
&S_\alpha = P_\mu \psi_\alpha^\mu + e(\gamma^5\pi_\mu^+)_\alpha \bar{X}^{\mu\prime} \\
&\bar{S}_\alpha = \bar{P}_\mu \psi_\alpha^{+\mu} + f(\pi_\mu\gamma^5)_\alpha X^{\mu\prime} \\
&U = \psi_\mu^+ \psi^\mu
\end{aligned}
\tag{11}
$$

where a, b, c, d, e and f are parameters to be determined. Equations (9) are recovered by setting $\pi_\alpha^\mu = \pi_\alpha^{+\mu} = 0$, of course.

With fundamental brackets

$$
\begin{aligned}
&\{X^\mu(\sigma), P^\nu(\sigma')\}_p = \eta^{\mu\nu}\delta(\sigma-\sigma') \\
&\{\bar{X}^\mu(\sigma), \bar{P}^\nu(\sigma')\}_p = \eta^{\mu\nu}(\delta(\sigma-\sigma') \\
&\{\psi_\alpha^\mu(\sigma), \pi_\beta^\nu(\sigma')\}_p = \eta^{\mu\nu}\delta_{\alpha\beta}\delta(\sigma-\sigma') \\
&\{\psi_\alpha^{+\mu}(\sigma), \pi_\beta^{+\nu}(\sigma')\}_p = \eta^{\mu\nu}\delta_{\alpha\beta}\delta(\sigma-\sigma')
\end{aligned}
\tag{12}
$$

it is straightforward to show that the algebra indeed closes for the following choice of parameters:

$$
a = b = e = f = 0 \qquad c = d = 1. \tag{13}
$$

Thus the null constraints are

$$H_\perp = P^\mu \bar{p}_\mu \tag{14a}$$

$$H_\parallel = P^\mu X'_\mu + \bar{p}^\mu \bar{X}'_\mu + \pi^\mu \psi'_\mu + \psi_\mu^{+\prime}\pi^{+\mu} \tag{14b}$$

$$S_\alpha = P_\mu \psi_\alpha^\mu \tag{14c}$$

$$\bar{S}_\alpha = \bar{P}_\mu \psi_\alpha^{+\mu} \tag{14d}$$

$$U = \psi_\mu^+ \psi^\mu. \tag{14e}$$

Therefore, the phase-space Lagrangian of the $N=2$ null spinning string is still given by (8); however, constraints are given by equations (14). In complete analogy to the tensile case, it is appropriate to choose the values of multipliers $\lambda_\perp$, $\lambda_\parallel$, ξ_α, $\bar{\xi}_\alpha$ and ω in order to fix the null gauge, in which the metric g_{mn} is conformally flat and gravitinos χ_m and $\bar{\chi}_m$ are equal to zero. The Lagrangian that we arrive at in the null gauge is:

$$L = \dot{X}^\mu \dot{\bar{X}}_\mu + \mathrm{i}\psi_\mu^+ \dot{\psi}^\mu + \mathrm{i}\dot{\psi}_\mu^+ \psi^\mu \tag{15}$$

and the constraints in this gauge are:

$$H_\perp = \dot{X}^\mu \dot{\bar{X}}_\mu \tag{16a}$$

$$H_\parallel = \dot{\bar{X}}^\mu X'_\mu + \dot{X}^\mu \bar{X}'_\mu + \mathrm{i}\psi_\mu^+ \psi^{\mu\prime} + \mathrm{i}\psi^\mu \psi_\mu^{+\prime} \tag{16b}$$

$$S_\alpha = \dot{\bar{X}}_\mu \psi_\alpha^\mu \tag{16c}$$

$$\bar{S}_\alpha = \dot{X}_\mu \psi_\alpha^{+\mu} \tag{16d}$$

$$U = \psi_\mu^+ \psi^\mu. \tag{16e}$$

The canonical quantization of the Lagrangian (15) is straightforward after fixing the light-cone gauge in order to retain the physical degrees of freedom. In the light-cone gauge ($X^+=(1/\sqrt{2\pi})p^+\tau$, $\bar{X}^+(1/\sqrt{2\pi})\bar{p}^+\tau$, $\psi^+_\alpha=0$, $(\psi^+_\alpha)^+=0$), the most general solution to the equations of motion does not involve any oscillators:

$$X^\mu(\tau,\sigma)=P^\mu(\sigma)\tau+Y^\mu(\sigma)$$
$$\bar{X}^\mu(\tau,\sigma)=\bar{P}^\mu(\sigma)\tau+\bar{Y}^\mu(\sigma) \tag{17}$$

$$\psi^\mu_\alpha(\tau,\sigma)=\psi^\mu_\alpha(\sigma)$$
$$\psi^{+\mu}_\alpha(\tau,\sigma)=\psi^{+\mu}_\alpha(\sigma) \tag{18}$$

and the constraints (16) become

$$(p^+\bar{p}^-+\bar{p}^+p^-)=\sqrt{2\pi}\,\vec{P}\cdot\vec{\bar{P}} \tag{19a}$$

$$(p^+Y^{-\prime}+\bar{p}^+\bar{Y}^{-\prime})=\sqrt{2\pi}\,[\vec{P}\cdot\vec{Y}'+\vec{\bar{P}}\cdot\vec{\bar{Y}}'+i(\vec{\psi}^+\cdot\vec{\psi}'+\vec{\psi}\cdot\vec{\psi}^{+\prime})] \tag{19b}$$

$$\psi^-_\alpha=\frac{1}{p^+}\sqrt{2\pi}\,\vec{P}\cdot\vec{\psi}_\alpha \tag{19c}$$

$$(\psi^+_\alpha)^-\frac{1}{\bar{p}^+}\sqrt{2\pi}\,\vec{\bar{P}}\cdot\vec{\psi}^+_\alpha \tag{19d}$$

$$\vec{\psi}^+\cdot\vec{\psi}=0 \tag{19e}$$

where $\vec{u}\cdot\vec{v}=u^Iv^I$, $I=1,2,\ldots,D-2$. The crucial point is that, unlike the $N=1$ case [13], the constraints (19) are unsolvable. In order to completely solve the constraints (19), we note that no kinetic term for the U(1) gauge field A_m appears in the original Lagrangian (1). This fact leads to the confinement of the U(1) charge. Therefore, we may introduce an ansatz that the field X^μ is invariant under U(1) gauge transformations, i.e. $\bar{X}^\mu=X^\mu$ and then $\bar{P}^\mu=P^\mu$. With help of this ansatz, we get the solution of the constraints (19):

$$P^-=\frac{\sqrt{2\pi}}{2p^+}\vec{P}^2 \tag{20a}$$

$$Y^{-1}=\frac{\sqrt{2\pi}}{p^+}\left(\vec{P}\cdot\vec{Y}'+\frac{i}{2}\vec{\psi}^+\cdot\vec{\psi}'+\frac{i}{2}\vec{\psi}\cdot\vec{\psi}^{+\prime}\right) \tag{20b}$$

$$\psi^-_\alpha=\frac{\sqrt{2\pi}}{p^+}\vec{P}\cdot\vec{\psi}_\alpha \tag{20c}$$

$$(\psi^+_\alpha)^-=\frac{\sqrt{2\pi}}{p^+}\vec{P}\cdot\vec{\psi}^+_\alpha \tag{20d}$$

$$\vec{\psi}^+\cdot\vec{\psi}=0. \tag{20e}$$

Now we may proceed to the canonical quantization and postulate the following fundamental brackets for the independent degrees of freedom,

$$[Y^I(\sigma),P^J(\sigma')]=i\delta^{IJ}\delta(\sigma-\sigma') \tag{21a}$$

$$\{\psi^I_\alpha(\sigma),\psi^{+J}_\beta(\sigma')\}=\delta_{\alpha\beta}\delta^{IJ}\delta(\sigma-\sigma'). \tag{21b}$$

It is convenient to expand the independent (transverse) degrees of freedom in Fourier series. In terms of modes, the fundamental brackets (21) become

$$[y_0^-, p^+] = -\mathrm{i} \tag{22a}$$

$$[y_m^I, p_n^J] = \mathrm{i}\delta^{IJ}\delta_{m+n,0} \tag{22b}$$

$$\{C_{\alpha,m}^I, d_{\beta,n}^J\} = \delta^{IJ}\delta_{\alpha\beta}\delta_{m+n,0}. \tag{22c}$$

For light-cone quantization, the quantum consistency amounts to requiring the Lorentz algebra to close. The generators of the Lorentz group are

$$\begin{aligned} M^{\mu\nu} &= \int \mathrm{d}\sigma\!: [P^\mu(\sigma)Y^\nu(\sigma) - P^\nu(\sigma)Y^\mu(\sigma) + \mathrm{i}(\psi_\alpha^{+\mu}(\sigma)\psi_\alpha^\nu(\sigma) - \psi_\alpha^{+\nu}(\sigma)\psi_\alpha^\mu(\sigma))]\!: \\ &= \sum_m :[p_{-m}^\mu y_m^\nu - p_{-m}^\nu y_m^\mu + \mathrm{i}(d_{\alpha,-m}^\mu C_{\alpha,m}^\nu - d_{\alpha,-m}^\nu C_{\alpha,m}^\mu)]: \end{aligned} \tag{23}$$

where the symbol : : denotes normal ordering. It is important to check that the operators in (23) really generate the Lorentz algebra. Most of the commutators can be checked straightforwardly and give the correct result. However, the commutator $[M^{I-}, M^{J-}]$ must be treated with care.

$$M^{I-} = :\left(p_0^I y_0^- + \frac{\mathrm{i}}{p^+}\sum_{n\neq 0}\frac{1}{n}p_n^I y_{-n} - \frac{1}{p^+}\sum_n p_n y_{-n}^I\right): \tag{24}$$

where we have taken the following redefinition for constraints:

$$y_n = \mathrm{i}\sum_m m\!:p_{n-m}^I y_m^I\!: + \tfrac{1}{2}\sum_{\alpha,m} m\!:(d_{\alpha,n+m}^I C_{\alpha,-m}^I + C_{\alpha,n+m}^I d_{\alpha,-m}^I)\!: \tag{25a}$$

$$p_n = p^+ p_n^- = \frac{1}{2}\sum_m :p_{-m}^I p_{n+m}^I: \tag{25b}$$

$$\phi_{\alpha,a} = p^+ C_{\alpha,a}^- = \sum_m :p_m^I C_{\alpha-a-m}^I: \tag{25c}$$

$$\bar\phi_{\alpha,a} = p^+ d_{\alpha,a}^- = \sum_m :p_m^I d_{\alpha,a-m}^I: \tag{25d}$$

$$T_n = \sum_m :d_{\alpha,m}^I C_{\alpha,n-m}^I:. \tag{25e}$$

The algebra formed by these constraints reads as

$$\begin{aligned} &[p_m, p_n] = 0 \qquad [y_m, p_n] = (n-m)p_{m+n} \\ &[\phi_{\alpha,a}, p_n] = [\bar\phi_{\alpha,a}, p_n] = [T_m, p_n] = 0 \\ &[y_m, y_n] = (n-m)y_{m+n} + A(n)\delta_{m+n,0} \\ &[y_n, \phi_{\alpha,a}] = (a-n/2)\phi_{\alpha,a+n} \qquad [y_n, \bar\phi_{\alpha,a}] = (a-n/2)\bar\phi_{\alpha,a+n} \\ &\{\phi_{\alpha,a}, \phi_{\beta,b}\} = \{\bar\phi_{\alpha,a}, \bar\phi_{\beta,b}\} = 0 \qquad \{\phi_{\alpha,a}, \bar\phi_{\beta,b}\} = 2\delta_{\alpha\beta}p_{a+b} \\ &[\phi_{\alpha,a}, T_n] = \phi_{\alpha,n+a} \qquad [\bar\phi_{\alpha,a}, T_n] = -\bar\phi_{\alpha,n+a} \\ &[T_m, T_n] = md\delta_{m+n,0} \qquad [y_m, T_n] = nT_{m+n} \end{aligned} \tag{26}$$

where $d = D-2$ and D is the dimension of spacetime, $A(m) = A^{\mathrm{B}}(m) + A^{\mathrm{F}}(m)$ is the ordering-prescription-dependent central term.

Under normal ordering, the physical Hilbert space is defined as being created out of a vacuum $|0\rangle$ annihilated by y_n^I, p_n^I, $\psi_{\alpha,n}^I$ and $\psi_{\alpha,n}^{+I}$, for n positive. Thus we have

$$y_n^I|0\rangle = p_n^I|0\rangle = \psi_{\alpha,n}^I|0\rangle = \psi_{\alpha,n}^{+I}|0\rangle = 0 \qquad n>0 \tag{27}$$

as well as

$$y_0|0\rangle = a|0\rangle \tag{28}$$

where a is the normal ordering constant.

Using the well known trick in computing anomalies [16], we obtain the following result:

$$A^{\mathrm{B}}(m) = \tfrac{1}{6}d(m^3 - m)$$

$$A^{\mathrm{F,R}}(m) = \tfrac{1}{6}d(m^3 + 2m)$$

$$A^{\mathrm{F,N}}(m) = \tfrac{1}{6}d(m^3 - m)$$

where A^{B} is the bosonic part of A, and superscripts R and N represent the R (Raymond) sector and N-S (Neveu–Schwarz) sectors respectively [13]. Then,

$$A^{\mathrm{R\text{-}R}} = \tfrac{1}{3}d\left(m^3 + \frac{m}{2}\right) \tag{29a}$$

$$A^{\mathrm{N\text{-}N}} = \tfrac{1}{3}d\ (m^3 - m) \tag{29b}$$

$$A^{\mathrm{N\text{-}R}} = A^{\mathrm{R\text{-}N}} = \tfrac{1}{3}d\left(m^3 - \frac{m}{4}\right). \tag{29c}$$

After a rather redious computation, we obtain for the Lorentz algebra

$$[M^{I-}, M^{J-}] = \frac{1}{(p^+)^2}\sum_{m>0}(p_{-m}^I p_m^J - p_{-m}^J p_m^I)\left(\frac{A(m)}{m^2} - \frac{2}{m}y_0\right). \tag{30}$$

Thus, inserting the central term (29) into (30), we finally obtain the critical dimension $D = d + 2 = 2$ and the values of y_0: $a_{\mathrm{R\text{-}R}} = a_{\mathrm{N\text{-}N}} = a_{\mathrm{R\text{-}N}} = a_{\mathrm{N\text{-}R}} = 0$.

On the basis of the previous work, we come to the conclusion that the $N=2$ extention of the null spinning string gives a highly symmetrical two-dimensional theory and an interesting generalization of the null super-Virasoro algebra. Unfortunately, it seems difficult to give the usual physical interpretation for this theory since the critical dimension is $D=2$ and there are not transverse excitations of strings in this dimension. Maybe it enters physics in some other unknown way. On generalizing our construction of $N=2$ null spinning strings to $N>2$ cases, by means of adding more fermionic constraints, it is amusing to note the the critical dimension corresponding to the $N=4$ case is $D=-2$, again in surprising agreement with the usual string theory, and this seems to have no sensible interpretation.

Acknowledgment

The authors would like to thank Dr Jian-Hui Dai for helpful discussions.

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